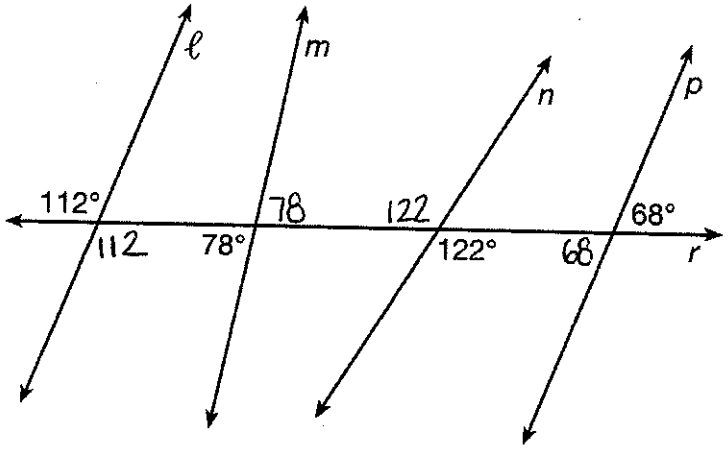
Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

1 In the diagram below, lines ℓ, m, n, and p intersect line r.

Use this space for computations.



112+68=180

Which statement is true?

(1) $\ell \| n$
(2) $\ell \| p$
(3) $m \| p$
(4) $m \| n$

2 Which transformation would *not* always produce an image that would be congruent to the original figure?

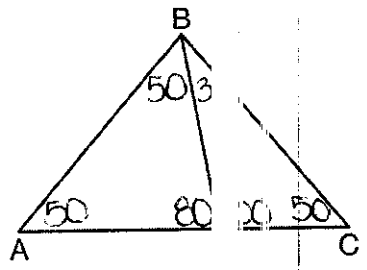
(1) translation
(2) dilation
(3) rotation
(4) reflection

3 If an equilateral triangle is continuously rotated around one of its medians, which 3-dimensional object is generated?

(1) cone
(2) pyramid
(3) prism
(4) sphere

Use this space for computations.

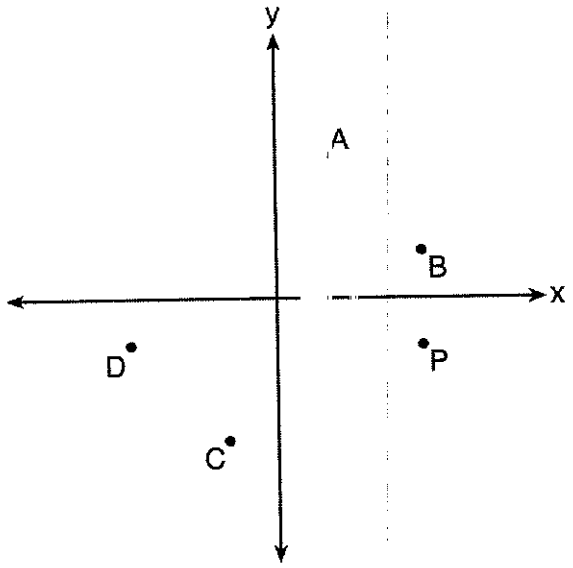
4 In the diagram below, $m\angle BDC = 100°$, $m\angle A = 50°$, and $m\angle DBC = 30°$.



Which statement is true?

(1) $\triangle ABD$ is obtuse.
(2) $\triangle ABC$ is isosceles.
(3) $m\angle ABD = 80°$
(4) $\triangle ABD$ is scalene.

5 Which point shown in the graph below is the image of point P after a counterclockwise rotation of 90° about the origin?



(1) A
(2) B
(3) C
(4) D

Use this space for computations.

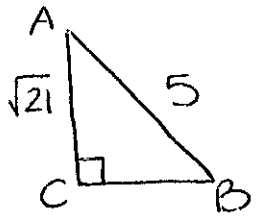
6 In $\triangle ABC$, where $\angle C$ is a right angle, $\cos A = \frac{\sqrt{21}}{5}$. What is $\sin B$?

● $\frac{\sqrt{21}}{5}$

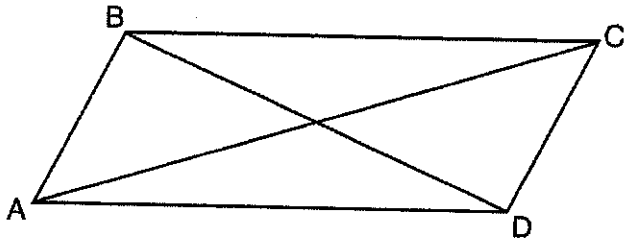
(2) $\frac{\sqrt{21}}{2}$

(3) $\frac{2}{5}$

(4) $\frac{5}{\sqrt{21}}$



7 Quadrilateral $ABCD$ with diagonals $\overline{AC}$ and $\overline{BD}$ is shown in the diagram below.



Which information is *not* enough to prove $ABCD$ is a parallelogram?

(1) $\overline{AB} \cong \overline{CD}$ and $\overline{AB} \parallel \overline{DC}$ 1 pair opp sides ∥ and ≅

(2) $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$ opp sides ≅

● $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \parallel \overline{AD}$

(4) $\overline{AB} \parallel \overline{DC}$ and $\overline{BC} \parallel \overline{AD}$ opp sides ∥

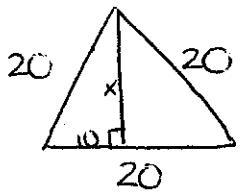
8 An equilateral triangle has sides of length 20. To the *nearest tenth*, what is the height of the equilateral triangle?

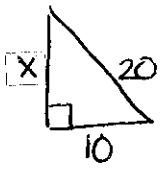
(1) 10.0

● 17.3

(2) 11.5

(4) 23.1





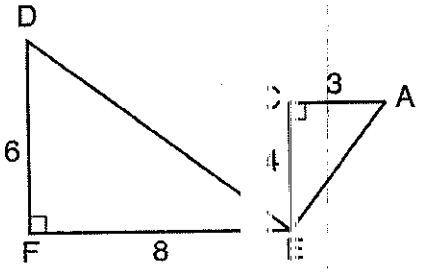
$x^2 + 10^2 = 20^2$

$x^2 + 100 = 400$

$\sqrt{x^2} = \sqrt{300}$

Use this space for computations.

9 Given: $\triangle AEC$, $\triangle DEF$, and $\overline{FE} \perp$ [illegible]



What is a correct sequence of sim [illegible]rity transformations that shows $\triangle AEC \sim \triangle DEF$?

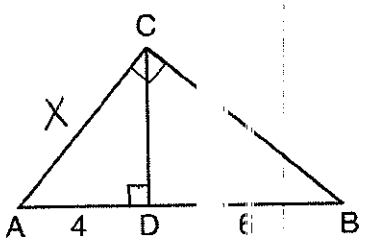
(1) a rotation of 180 degrees abou [illegible]oint E followed by a horizontal translation

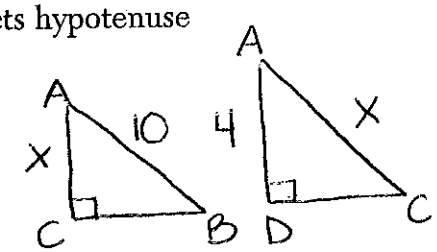
(2) a counterclockwise rotation [illegible]f 90 degrees about point E followed by a horizontal trans [illegible]tion

(3) a rotation of 180 degrees abc [illegible] point E followed by a dilation with a scale factor of 2 center [illegible]t point E

(4) a counterclockwise rotation [illegible]f 90 degrees about point E followed by a dilation with a s [illegible]le factor of 2 centered at point E

10 In the diagram of right triangle A [illegible] C, $\overline{CD}$ intersects hypotenuse $\overline{AB}$ at D.





$\frac{X}{10} = \frac{4}{X}$

$\sqrt{X^2} = \sqrt{40}$

$\sqrt{4}\sqrt{10}$

$2\sqrt{10}$

If $AD = 4$ and $DB = 6$, which le [illegible]th of $\overline{AC}$ makes $\overline{CD} \perp \overline{AB}$?

(1) $2\sqrt{6}$ (3) $2\sqrt{15}$

(2) $2\sqrt{10}$ (4) $4\sqrt{2}$

Use this space for computations.

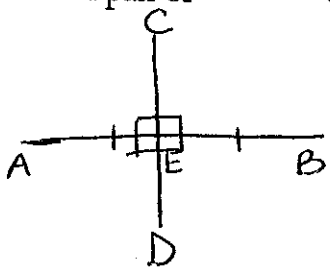
11 Segment CD is the perpendicular bisector of $\overline{AB}$ at E. Which pair of segments does *not* have to be congruent?

(1) $\overline{AD}, \overline{BD}$

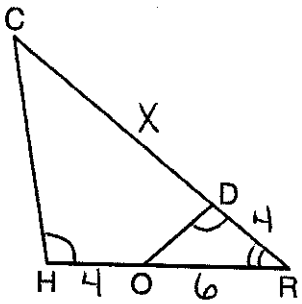
(2) $\overline{AC}, \overline{BC}$

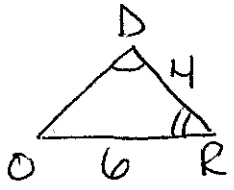
(3) $\overline{AE}, \overline{BE}$

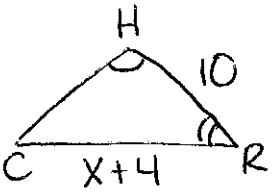
● $\overline{DE}, \overline{CE}$



12 In triangle CHR, O is on $\overline{HR}$, and D is on $\overline{CR}$ so that $\angle H \cong \angle RDO$.







$\frac{4}{6} = \frac{10}{X+4}$

$4X+16=60$

$-16 \quad -16$

$4X=44$

$X=11$

If $RD = 4$, $RO = 6$, and $OH = 4$, what is the length of $\overline{CD}$?

(1) $2\frac{2}{3}$

(2) $6\frac{2}{3}$

● 11

(4) 15

13 The cross section of a regular pyramid contains the altitude of the pyramid. The shape of this cross section is a

(1) circle

(2) square

● triangle

(4) rectangle

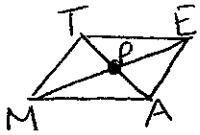
14 The diagonals of rhombus $TEAM$ intersect at $P(2,1)$. If the equation of the line that contains diagonal $\overline{TA}$ is $y = -x + 3$, what is the equation of a line that contains diagonal $\overline{EM}$?

● $y = x - 1$

(2) $y = x - 3$

(3) $y = -x - 1$

(4) $y = -x - 3$



$m=-1$

$\perp m=1$

$y-y=m(x-x)$

$y-1=1(x-2)$

$y-1=x-2$

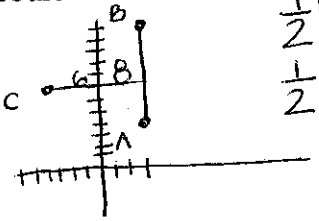
$+1 \quad +1$

$y=x-1$

Use this space for computations.

15 The coordinates of vertices A and B of $\triangle ABC$ are $A(3,4)$ and $B(3,12)$. If the area of $\triangle ABC$ is 24 square units, what could be the coordinates of point C?

(1) (3,6)
(2) (8,−3)
(3) (−3,8)
(4) (6,3)



$\frac{1}{2}bh = 24$
$\frac{1}{2}(8)(h) = 24$
$4h = 24$
$h = 6$

$(-\frac{4}{2})^2 = (-2)^2 = 4 \quad (\frac{8}{2})^2 = (4)^2 = 16$

16 What are the coordinates of the center and the length of the radius of the circle represented by the equation $x^2 + y^2 - 4x + 8y + 11 = 0$?

(1) center $(2,-4)$ and radius 3
(2) center $(-2,4)$ and radius 3
(3) center $(2,-4)$ and radius 9
(4) center $(-2,4)$ and radius 9

$x^2 - 4x + 4 + y^2 + 8y + 16 = -11 + 4 + 16$
$(x-2)^2 + (y+4)^2 = 9$
$C = (2,-4) \quad \sqrt{r^2} = \sqrt{9}$
$r = 3$

17 The density of the American white oak tree is 752 kilograms per cubic meter. If the trunk of an American white oak tree has a circumference of 4.5 meters and the height of the trunk is 8 meters, what is the approximate number of kilograms of the trunk?

(1) 13
(2) 9694
(3) 13,536
(4) 30,456

$V = \pi r^2 h$
$C = \pi d$
$\frac{4.5}{\pi} = \frac{\pi d}{\pi}$
$1.432394488 = d$
$\frac{}{2}$
$.7161972439 = r$

$V = \pi(.71619)^2(8)$
$V = 12.89155039$

$D = \frac{M}{V}$
$\frac{752}{1} = \frac{M}{12.89155503}$
$M = 9694$

18 Point P is on the directed line segment from point $X(-6,-2)$ to point $Y(6,7)$ and divides the segment in the ratio 1:5. What are the coordinates of point P?

Use this space for computations.

$X(-6,-2) \xrightarrow[12(\frac{1}{6})\ 9(\frac{1}{6})]{T\langle 12,9\rangle} Y(6,7)$

$X(-6,-2) \xrightarrow{T\langle 2,1.5\rangle} P(-4,-.5)$

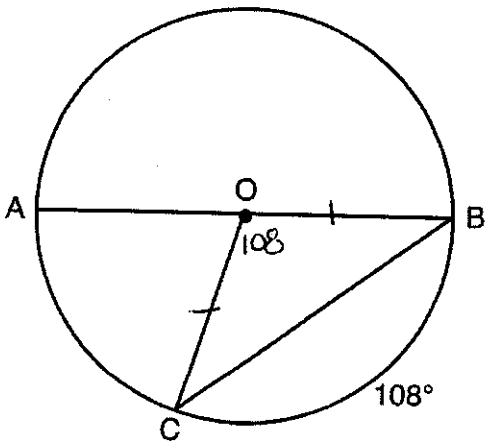
(1) $(4,5\frac{1}{2})$

(3) $(-4\frac{1}{2},0)$

(2) $(-\frac{1}{2},-4)$

● $(-4,-\frac{1}{2})$

19 In circle O, diameter $\overline{AB}$, chord $\overline{BC}$, and radius $\overline{OC}$ are drawn, and the measure of arc BC is 108°.



Some students wrote these formulas to find the area of sector COB:

Amy $\frac{3}{10} \cdot \pi \cdot (BC)^2$

Beth $\frac{108}{360} \cdot \pi \cdot (OC)^2$

Carl $\frac{3}{10} \cdot \pi \cdot (\frac{1}{2}AB)^2$

Dex $\frac{108}{360} \cdot \pi \cdot \frac{1}{2}(AB)^2$

$\frac{n}{360} \cdot \pi r^2$

$\frac{108}{360} \cdot \pi r^2$

Which students wrote correct formulas?

(1) Amy and Dex

(3) Carl and Amy

● Beth and Carl

(4) Dex and Beth

Use this space for computations.

20 Tennis balls are sold in cylindrica cans with the balls stacked one on top of the other. A tennis ball as a diameter of 6.7 cm. To the *nearest cubic centimeter*, what is e minimum volume of the can that holds a stack of 4 tennis balls

(1) 236
(3) 564
(2) 282
(4) 945

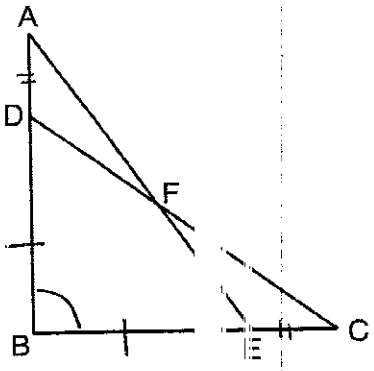
$D=6.7$
$r=3.35$
$h=4D$
$4(6.7)$
26.8
$V=\pi r^2 h$
$\pi(3.35)^2(26.8)$
945

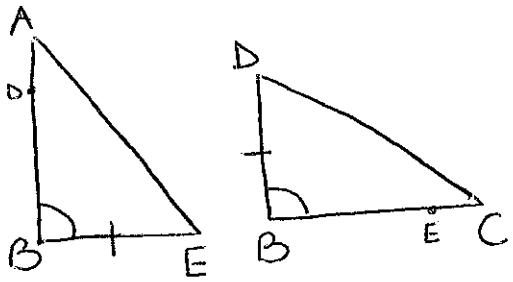
21 Line segment $A'B'$, whose endpoints are (4,−2) and (16,14), is the image of $\overline{AB}$ after a dilation of $\frac{1}{2}$ centered at the origin. What is the length of $\overline{AB}$?

(1) 5
(3) 20
(2) 10
(4) 40

$A(8,4)$
$B(32,28)$
$\sqrt{(8-32)^2+(4-28)^2}$
$\sqrt{(-24)^2+(-32)^2}$
$\sqrt{1600}$

22 Given: $\triangle ABE$ and $\triangle CBD$ shown in the diagram below with $\overline{DB} \cong \overline{BE}$



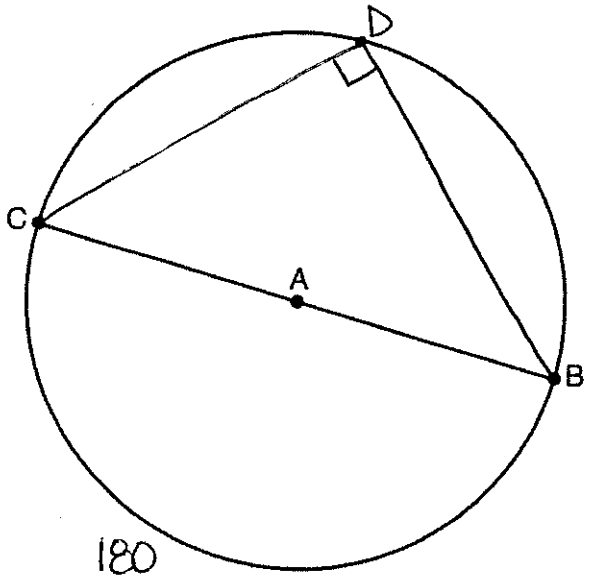


Which statement is needed to prove $\triangle ABE \cong \triangle CBD$ using only SAS ≅ SAS?

(1) $\angle CDB \cong \angle AEB$
(3) $\overline{AD} \cong \overline{CE}$
(2) $\angle AFD \cong \angle EFC$
(4) $\overline{AE} \cong \overline{CD}$

23 In the diagram below, $\overline{BC}$ is the diameter of circle A.

Use this space for computations.



Point D, which is unique from points B and C, is plotted on circle A. Which statement must always be true?

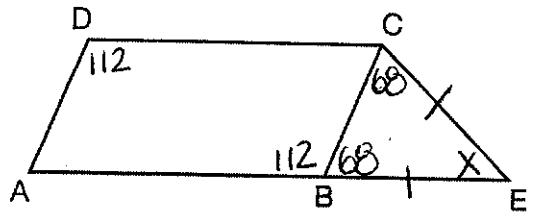
● $\triangle BCD$ is a right triangle.

(2) $\triangle BCD$ is an isosceles triangle.

(3) $\triangle BAD$ and $\triangle CBD$ are similar triangles.

(4) $\triangle BAD$ and $\triangle CAD$ are congruent triangles.

24 In the diagram below, $ABCD$ is a parallelogram, $\overline{AB}$ is extended through B to E, and $\overline{CE}$ is drawn.



180-112=68

X+68+68=180

X+136=180

X=44

If $\overline{CE} \cong \overline{BE}$ and $m\angle D = 112°$, what is $m\angle E$?

● 44°

(2) 56°

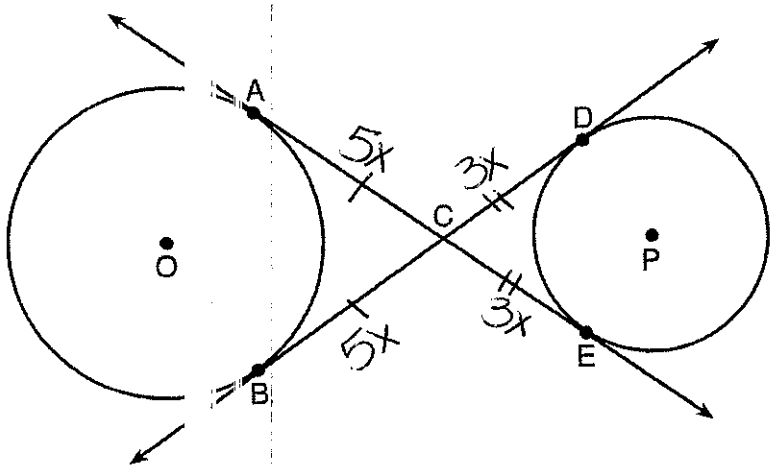
(3) 68°

(4) 112°

Part II

Answer all 7 questions in thi part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, incl ing appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information rovided for each question to determine your answer. Note that diagrams are not necessaril drawn to scale. For all questions in this part, a correct numerical answer with no work hown will receive only 1 credit. All answers should be written in pen, except for graphs d drawings, which should be done in pencil. [14]

25 Lines AE and BD are tangent to circles O and P at A, E, B, and D, as shown in the diagram below. If $AC:CE = 5:3$, and $BD = 56$, determine and state the length of $\overline{CD}$.



$5x + 3x = 56$

$8x = 56$

$x = 7$

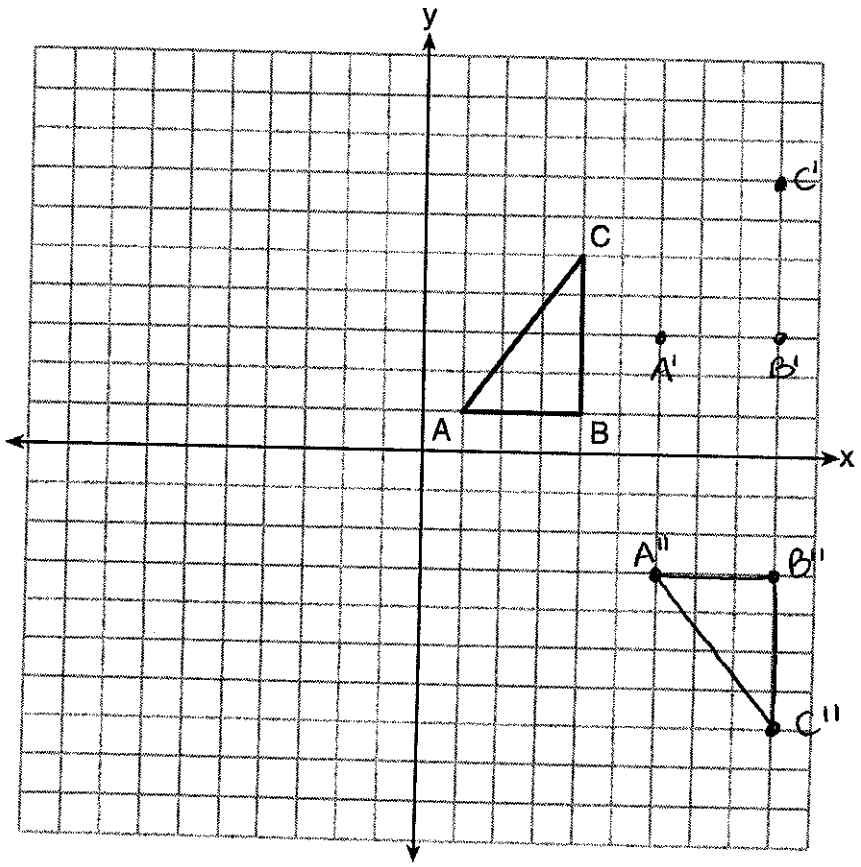
$CD = 3x$

$3(7)$

21

26 In the diagram below, $\triangle ABC$ has coordinates $A(1,1)$, $B(4,1)$, and $C(4,5)$. Graph and label $\triangle A''B''C''$, the image of $\triangle ABC$ after the translation five units to the right and two units up followed by the reflection over the line $y = 0$.

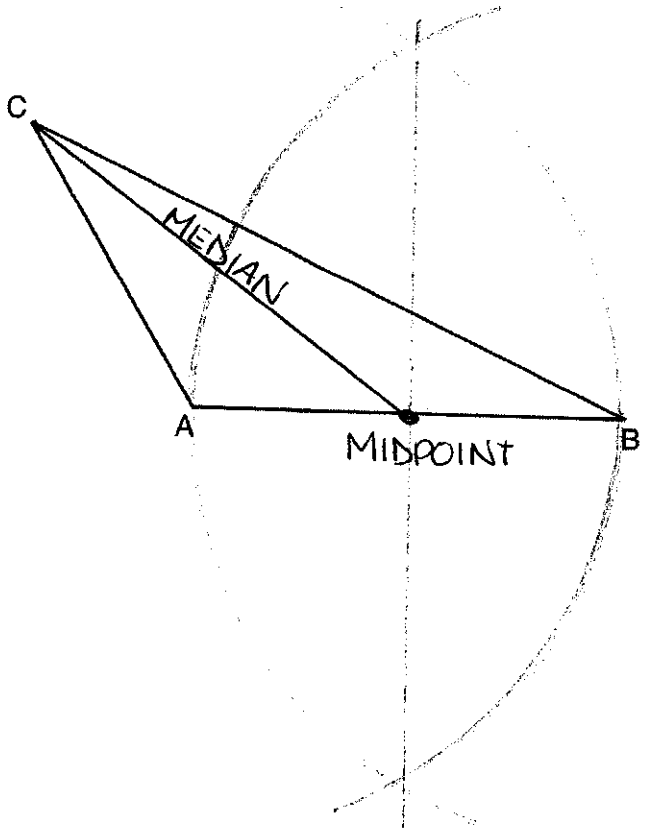
(x-axis)



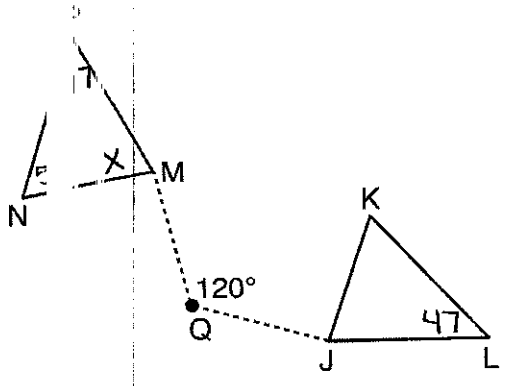
27 A regular hexagon is rotated in a counterclockwise direction about its center. Determine and state the minimum number of degrees in the rotation such that the hexagon will coincide with itself.

$\frac{360}{6} = 60$

28 In the diagram of $\triangle ABC$ shown below, use a compass and straightedge to construct the median to $\overline{AB}$. [Leave all construction marks.]



29 Triangle MNP is the image of tr ngle JKL after a 120° counterclockwise rotation about point Q. If the measure of angle L is 47 and the measure of angle N is 57°, determine the measure of angle M. Explain how you arriv at your answer.



$X+57+47=180$

$X+104=180$

$-104\ \ -104$

$X=76$

A rotation is a rigid motion that preserves angle measure. Angle L rotates onto Angle P.

30 A circle has a center at $(\overset{h}{1},\overset{k}{-2})$ and radius of $\overset{r}{4}$. Does the point $(\overset{x}{3.4},\overset{y}{1.2})$ lie on the circle? Justify your answer.

$$(x-h)^2+(y-k)^2=r^2$$

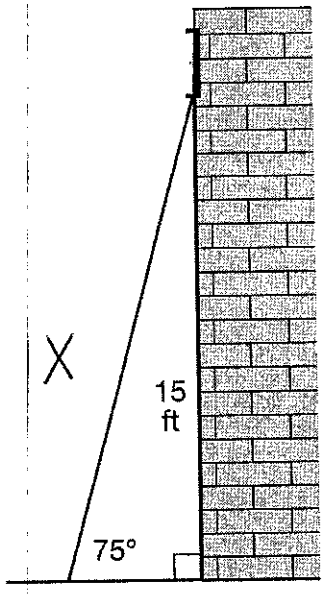
$$(3.4-1)^2+(1.2--2)^2=4^2$$

$$(2.4)^2+(3.2)^2=16$$

$$5.76+10.24=16$$

$$16=16\ \checkmark \quad \text{Yes}$$

31 In the diagram below, a windov of a house is 15 feet above the ground. A ladder is placed against the house with its base at an an e of 75° with the ground. Determine and state the length of the ladder to the *nearest tenth of a* oot.



$\frac{\sin 75}{1} = \frac{15}{X}$

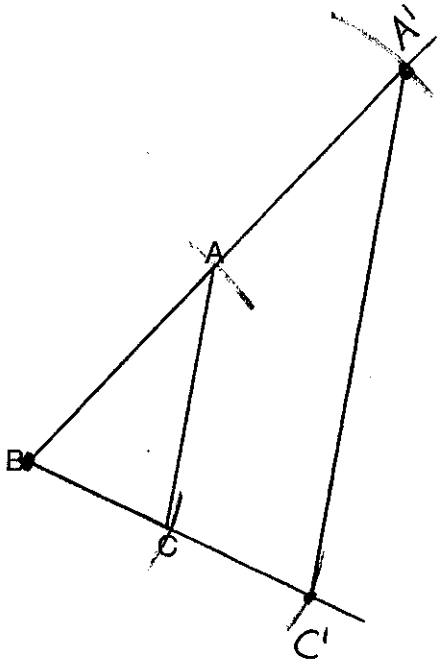
$X \cdot \frac{\sin 75}{\sin 75} = \frac{15}{\sin 75}$

$X = 15.5$

Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

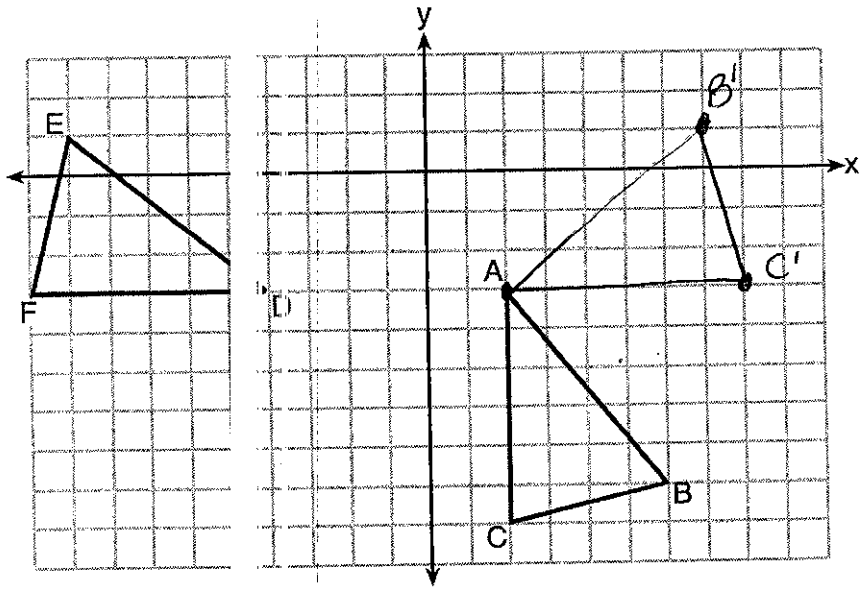
32 Using a compass and straightedge, construct and label $\triangle A'B'C'$, the image of $\triangle ABC$ after a dilation with a scale factor of 2 and centered at B. [Leave all construction marks.]



Describe the relationship between the lengths of $\overline{AC}$ and $\overline{A'C'}$.

the length of $\overline{A'C'}$ is twice the length of $\overline{AC}$

33 The grid below shows $\triangle ABC$ a l $\triangle DEF$.



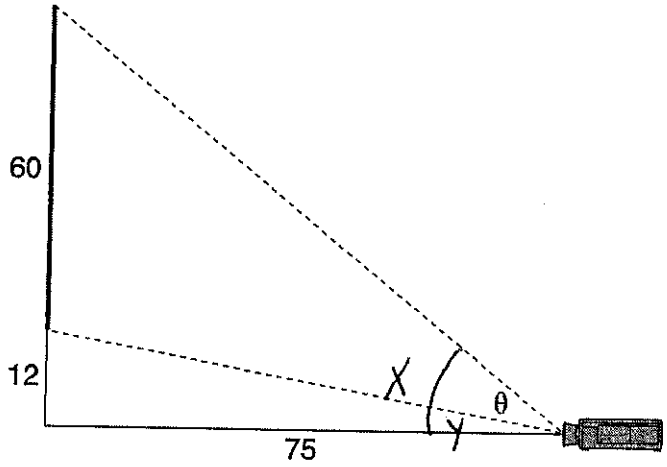
Let $\triangle A'B'C'$ be the image of $\triangle ABC$ after a rotation about point A. Determine and state the location of B' if the location of point C' is (8,–3). Explain your answer.

B'(7,1) $\triangle ABC$ rotated 90° counterclockwise around Point A

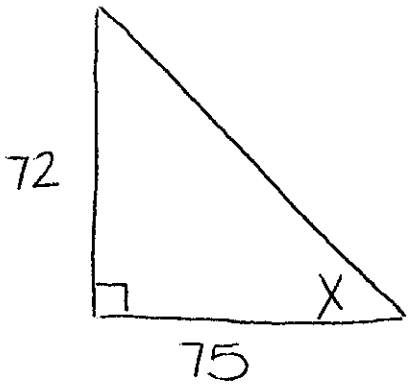
Is $\triangle DEF$ congruent to $\triangle A'B'C'$? Explain your answer.

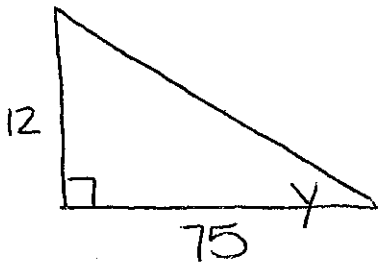
Yes $\triangle A'B'C'$ reflects over line $x=-1$ and maps onto $\triangle DEF$. A line reflection is a rigid motion that preserves size and angle measure.

34 As modeled below, a movie is projected onto a large outdoor screen. The bottom of the 60-foot-tall screen is 12 feet off the ground. The projector sits on the ground at a horizontal distance of 75 feet from the screen.



Determine and state, to the *nearest tenth of a degree*, the measure of θ, the projection angle.





$\tan X = \frac{72}{75}$

$\tan Y = \frac{12}{75}$

$X = 43.83086067$

$Y = 9.090276921$

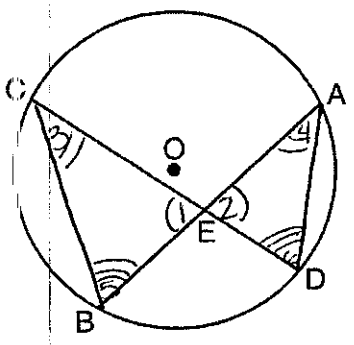
$43.83086067 - 9.090276921$

$\boxed{34.7}$

Part IV

Answer the 2 questions in this **art. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil.** [12]

35 Given: Circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E

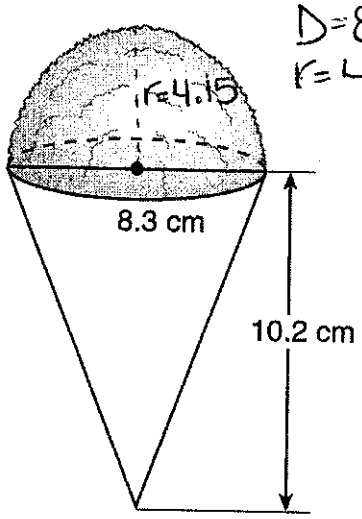


Theorem: If two chords intersect in a circle, the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.

Prove this theorem by proving $AE \cdot EB = CE \cdot ED$.

S	R
①Chords AB & CD	①Given
②$\angle 1 \cong \angle 2$	②Intersecting lines form $\cong$ vertical $\angle$s
③CB & AD are drawn	③Auxiliary lines can be drawn
④$\angle 3 \cong \angle 4$ $\angle 5 \cong \angle 6$	④Inscribed $\angle$s that intercept the same arc are $\cong$
⑤$\triangle ECB \sim \triangle EAD$	⑤AA ~
⑥$\frac{AE}{CE} = \frac{ED}{EB}$	⑥Corresponding sides of ~$\triangle$s are in proportion
⑦$AE \cdot EB = CE \cdot ED$	⑦In a proportion, cross products are equal

36 A snow cone consists of a paper cone completely filled with shaved ice and topped with a hemisphere of shaved ice, as shown in the diagram below. The inside diameter of both the cone and the hemisphere is 8.3 centimeters. The height of the cone is 10.2 centimeters.



D=8.3
r=4.15

The desired density of the shaved ice is 0.697 g/cm^3, and the cost, per kilogram, of ice is \$3.83. Determine and state the cost of the ice needed to make 50 snow cones.

$V_{cone} + V_{hemisphere}$

$\frac{1}{3}\pi r^2 h + \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$\frac{1}{3}\pi(4.15)^2(10.2) + \frac{1}{2}\left(\frac{4}{3}\pi\right)(4.15)^3$

333.6541568 cm^3

$D = \frac{M}{V}$

$\frac{.697}{1} = \frac{M}{333.6541568}$

$M = \frac{232.5569473\ g}{1000}$

$M = .2325569473\ kg$

$X \quad 50$

11.62784737

$X \quad 3.83$

\$44.53